**Weak Disorder**

Now let’s see if we can do a 1D δ potential exactly, within a certain self-energy approximation schemea.

**1D δ Potential via T-matrix Approximation**

Consider once again scattering of noninteracting electrons by the zero-range impurity potential.



Recall in the first file at the top of the folder, we said that in the low impurity density limit, the self-energy could be approximated by the series:



and was given by:



where the T-matrix is:



Well, in the Quantum Mechanics/Time Dependent/Constant Perturbation Examples file, we worked out the delta function potential problem exactly, by solving for the T-matrix. So I guess we’ll repeat those steps here, with the appropriate updates. So we start with the recursion relation:



(these are all operators, but MathType doesn’t let me indicate that aesthetically) And now let’s put this in a basis. We’ll dot both sides with <k| and |k´>, and insert resolutions of identity as needed,



Now the nice thing about the δ potential is that



So we have:



Pondering this equation a bit, we might observe that on the right hand side, the k1 index of Tk1k´ is integrated over, meaning the result of the integral will only depend on the k´ index. Thus considering Tkk´(iωn) is equal to this thing, that means that Tkk´(iωn) can at most depend on its k´ index alone. Given this, we can eliminate the first index of T from its argument, and write,



which makes it now trivial to solve for T,



But we’ll now observe that the RHS doesn’t even depend on k´. So really, T doesn’t have any index dependence at all – that’s what’s nice about the delta function potential. So we have, changing variable of integration from k1 -> k:



Let’s now do this integral,



If ωn > 0, we can factor the denominator, and close the contour, up, say, to get:



And if ω < 0, then we do similarly, and close the contour up, say,



If we agree that √(-|ω|) = i√ω, then we can combine these two results to simply say,



And from this, we get the self-energy:



So then we see,



and then from:



the Green’s function follows:



The retarded GF would be (I guess we can say isgn(ωn) = sgn(iωn) -> sgn(ω)?):



[remember, FWIW, that the chemical potential μ is implicitly absorbing the term niVi(q=0) = niV0, and so if we put that back in, we’d have ξk -> ξk + niaV0, which would cancel the other aV0ni in the denominator – confusing I know – but I’m just going to leave the niVi(q=0) term implicit in μ]. Let’s take a look at the small V0 limit. Then,



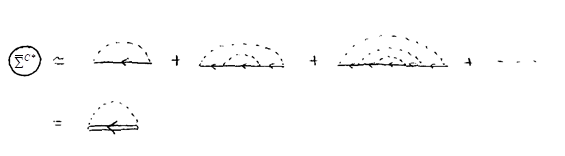
Poles of the GF determine the excitations. So let’s look at:



Or not.

**Self-Consistent Born Approximation with δ potential**

So in the previous file we introduced the self-consistent Born approximation for the self-energy:



And of course this could be written out as:



with the solution,



And according to our self-consistent self-energy diagram above, we also have:



Now let’s work this out using the δ function potential,



Then we have:



So we have two equations and two unknowns. We can fill our expression for C\* into the self-energy equation, and get:



Might observe that if we neglect the Σ in the denominator, then we get our first order self-energy approximation from a previous example. We’ll note that we can change variables in the integral q + k → q´, which we’ll just designate as q again, which will give us:



But this means that C\* doesn’t depend on k since the RHS doesn’t. So now we can say,



and we can do this integral, kind of. First I guess we’ll change variables to ε,



where ρ(ε) is the (spin-less) density of states. But it looks like we’ll run into some integrability issues again. Let’s separate into real and imaginary parts,



Can see the real part will not converge, but we’ll argue like we did when analyzing the first order term, that if we had a more realistic, i.e., damped V(q), the real part would be insignificant, especially for k near the Fermi surface. But like before, the imaginary part does converge, so we’ll focus on that.



Going to go the cheap route and approximate the density of states as constant with the value equal to that where the integrand is largest, which would be at ε = μ, i.e., at the Fermi surface. So we’ll approximately have:



and so we come to,



which is, underwhelmingly, what we had before, at first order. So maybe we get zero for all diagrams higher than 2nd order when using a δ function potential (at least with our approximation of constant density of states over the entire range, and that we could extend μ´ to ∞ in that integral)?